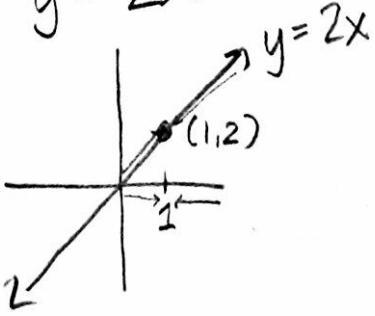
Lesson 2: Limits, Part 1

Idea: The limit of a function $f(x)$ is the value $f(x)$ approaches as x approaches a given value.

Ex 1 $y = 2x$



$$\lim_{x \to 1} 2x = 2$$

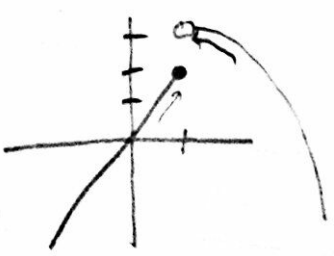
One Sided Limits

A one-sided limit of $f(x)$ is the value $f(x)$ approaches as x approaches a given value from a chosen side.

Ex 1 $\lim_{x \to 1^-} 2x = 2$ (approach 1 from left)

$\lim_{x \to 1^+} 2x = 2$ (approach 1 from right)

Ex 2 Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$:



$$f(x) = \begin{cases} 2x, & x \leq 1 \\ 4 - x^2, & x > 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = 2$

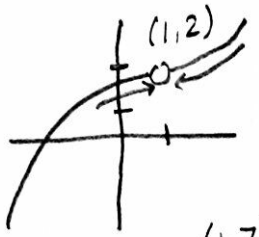
$\lim_{x \to 1^+} f(x) = 3$

$\lim_{x \to 1} f(x)$ DNE

Def If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$, then $\lim_{x \to c} f(x) = L$.

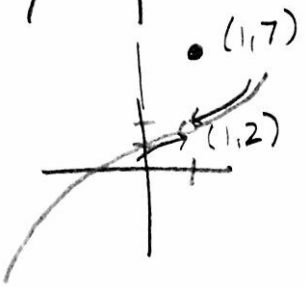
If $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$, then the limit does not exist (DNE)

Ex 3



$\lim_{x \to 1} f(x) = 2$

Ex 4



$\lim_{x \to 1} f(x) = 2$

Evaluating Limits Numerically

Ex 5 $\lim_{x \to 0} \frac{\sin x}{x} = ?$

x	-1	-.1	-.01	0	.01	.1	1
$\frac{\sin x}{x}$	.0175	.998	.999...	?	.999...	.998	.8415

↑ undefined

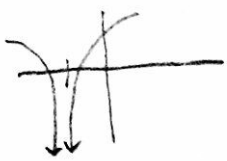
$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$

$\lim_{x \to 0^-} \frac{\sin x}{x} = 1$

$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$

Ex 6 $\lim_{x \to -1} \frac{-3}{(x+1)^2}$

x	-1.01	-1.001	-1			
$\frac{-3}{(x+1)^2}$	-30,000	-3 mil	?			



$\lim_{x \to -1^-} f(x) = -\infty$ $\quad \lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to -1} f(x) = -\infty$

Careful! If the limit exists, we can find it numerically. But sometimes the limit doesn't exist.

Ex 7 $\lim_{x \to 0+} \sin\left(\frac{\pi}{x}\right)$

x	0		.01	.1
$\sin\left(\frac{\pi}{x}\right)$	?		0	$\sin(10x) = 0$

But the limit doesn't exist (graph it to see why!).